Area

Approximations with Riemann Sums

The area under a curve can be approximated through the use of rectangles or Riemann sums according to the formula $\sum_{k=1}^{n} f\left(x_k\right)\Delta x_k$.

We will use the following conventions to compute the rectangular sums

Let $L_n =$ Sum of *n* rectangles using the left-hand *x*-coordinate of each interval to find the height of the rectangle.

Let $R_n =$ Sum of *n* rectangles using the right-hand *x*-coordinate of each interval to find the height of the rectangle.

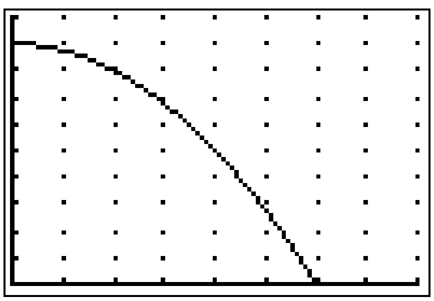
Let $M_n =$ Sum of *n* rectangles using the midpoint *x*-coordinate of each interval to find the height of the rectangle.

Example 1. Let $f(x) = 9 - x^2$ on [0, 3].

a) If all intervals are the same width and there are six rectangles/trapezoids, what is the value of Δx_k ?________ Use this value to compute the following.

Draw a sketch of the function along with the indicated rectangles

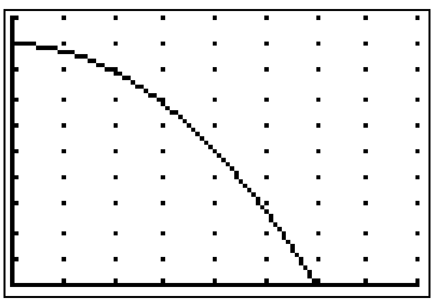
L_6 =



Sum =

Overestimate or Underestimate?

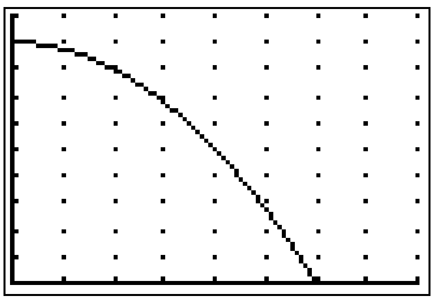
R_6 =



Sum =

Overestimate or Underestimate?

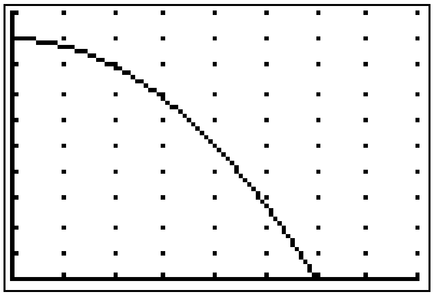
M_6 =



Sum =

Trapezoidal sums according to the formula: $\sum_{k=1}^{n} \frac{1}{2}\left[f\left(x_k\right) + f\left(x_{k+1}\right)\right] \Delta x_k .$

Let $T_n =$ Sum of *n* trapezoids using <u>consecutive *x*-coordinates</u> of each interval to find the base lengths of the trapezoid.

$T_6 =$  Sum =

Overestimate or Underestimate?

Example 2. Let $f(x) = 3^x$ on [-1, 3]

a) Find

$L_4 =$

$R_4 =$

$M_4 =$

$T_4 =$

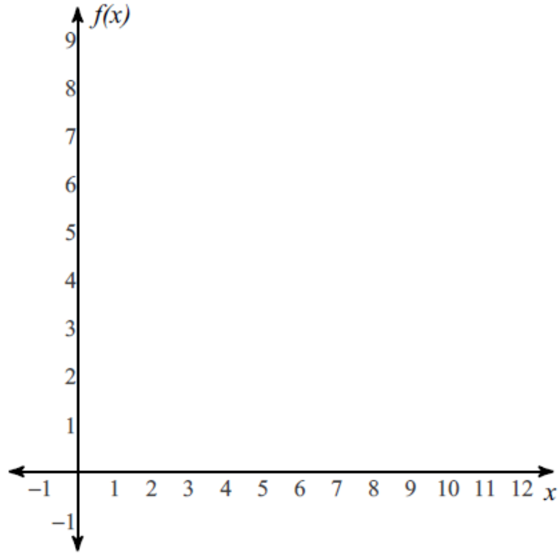
Table Problems – Approximations

Although the majority of the problems that we work with in most math classes involve equations, it would be very hard to find actual equations in a real-life setting. Many real-life settings involve data collection and the data is sorted in tables. If enough data is collected, then accurate approximations can be made for the real-life scenario. We will examine simple table problems and the Calculus that can be applied to this data. This is called solving problems "numerically".

Example 1

$f(x)$ is a differentiable function for all *x*. Selected values of $f(x)$ are given in the table below.

x	0	6	8	9	12
$f(x)$	7	6	8	5	7



a) Use the data in the table to approximate the area under $f(x)$ using a left Riemann sum with 4 subintervals.

b) Use the data in the table to approximate the area under $f(x)$ using a right Riemann sum with 4 subintervals.

c) Use the data in the table to approximate the area under $f(x)$ using a trapezoidal sum with 4 subintervals.

Example 2

$L(t)$ is a differentiable function that is decreasing for all *x*. Selected values of $L(t)$ are shown in the table below.

t	0	4	6	9	11	15
$L(t)$	30	24	17	12	10	9

a) Use the data in the table to approximate the area under $L(t)$ over the interval $[0,15]$ using a right Riemann sum with 5 subintervals. Is the approximation an overestimate or underestimate? Explain.

b) Use the data in the table to approximate $L'(10)$. Show the work that leads to your answer.

c) Calculate the average rate of change of $L(t)$ over the interval $[0,15]$.

d) Evaluate $\int_0^{11} L'(t)\,dt$